2001 closed book. open note.

① similar to 1

② 3layer problem (R.T etc--)

③ ~~TotMN~~ TE_{mn} mode.

ECE 550
First Exam
Dr. John F. Vetelino
March 23, 1999
Closed Notes

1. A good conducting medium is characterized by the following material parameters which are all constant with respect to time and position

Dielectric constant $= \varepsilon_o$

Electrical conductivity $= \sigma_o$

Permeability $= \mu_o$

With respect to a rectangular coordinate system in this medium, an electromagnetic wave is traveling in the <104> direction and is polarized so that the electric field is in the positive y-direction. Obtain an expression for the following quantities.

a. The magnetic field.

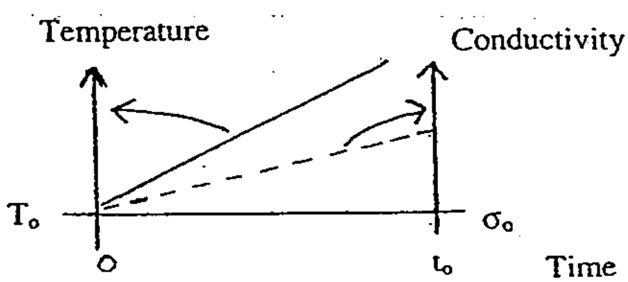
b. The attenuation associated with this wave.

c. Expressions for the phase and group velocities.

d. Is this medium dispersive? Justify your answer.

e. The Poynting vector.

2. Light is travelling through a semiconducting medium with finite conductivity and a constant dielectric and permeability constant. It is however observed that when the temperature increases the conductivity also increases which is normal behavior for a semiconductor. If the temperature increases with time as shown below, the conductivity varies as indicated



$T = at + T_o$

and $\qquad 0 \le t \le t_o$

$\sigma = bt + \sigma_o$

where

a = constant

b = constant

t = time

T = temperature

T_o = reference temperature

σ_o = reference conductivity

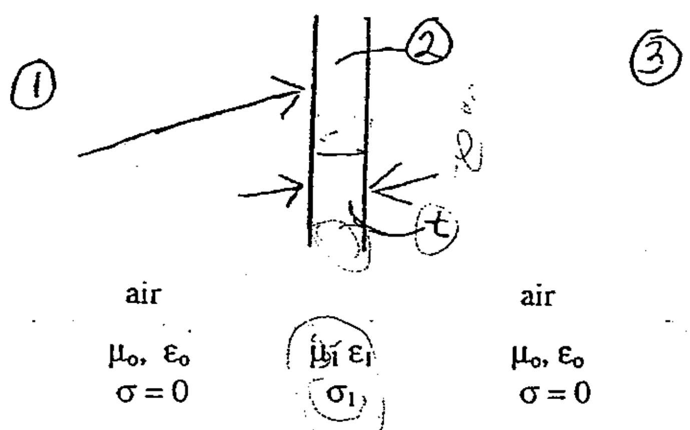
t_o = the time interval in which the temperature increases

a. Develop the appropriate time dependent wave equation for the electric field which must be solved in order to describe propagating electromagnetic waves in the medium in the time interval $0 \le t \le t_o$. Assume that there are no stationary charges (ρ) in the medium.

b. If the field quantities are assumed to vary harmonically with time, what is the resulting wave equation?

c. Can this wave equation be solved? If so, what is the solution.

3. For the geometry shown below electromagnetic waves are incident from air onto a thin plate with finite conductivity.

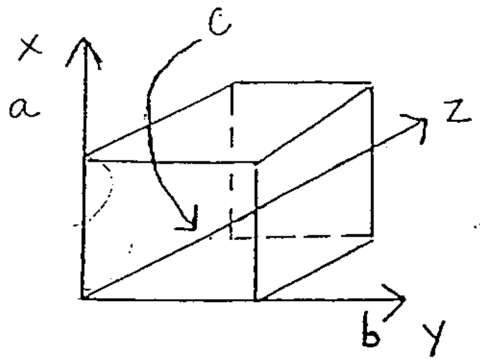


a. If the plate has a thickness of t and the electromagnetic waves are incident normally on the plate, obtain an expression for the reflection coefficient.

b. Repeat part a for nonnormal incidence.

c. If the plate is a perfect conductor what are the electromagnetic fields in medium 3?

4. A resonator is made by placing a conducting sheet at $z = c$ in the rectangular guide shown below.



a. If the E-field is oriented in the y-direction in the resonator obtain an expression for the cutoff frequencies for the fundamental and higher order modes in the resonator.

b. If each wall has the same losses what dimensions of the resonator will maximize the Q.

ECE 550
First Exam
Closed Notes and Books
March 9, 2000

1. If the source of electromagnetic energy is due to a current source, what is the equation for the time-average power balance for the following media.

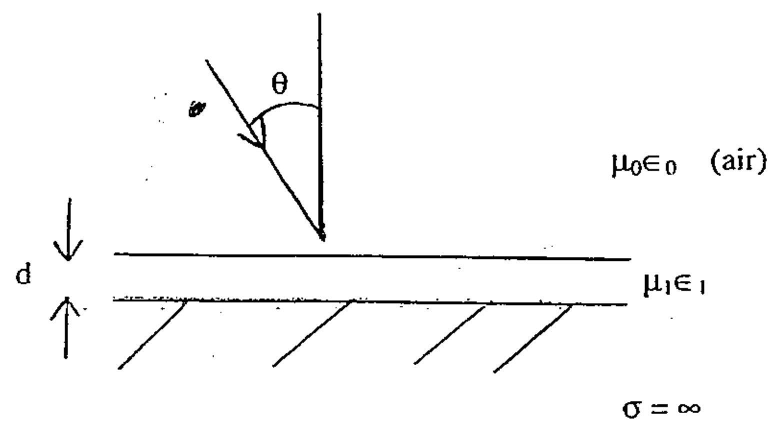
   a. Perfect dielectric medium
   b. A medium which has only dielectric losses
   c. A good conducting medium
   d. A medium which is conducting and has dielectric losses but no magnetic losses
   e. What is the quality factor for each of media described in a-d?

2. A medium is characterized as follows:

   - μ and ϵ are constants with respect to space and time
   - the charge density, ρ, is finite but not a function of position or time
   - σ is constant with respect to time but is a function of position

   a. Obtain the wave equations for the electric and magnetic fields.
   b. Can these wave equations be solved and if so what is the solution.

3. A thin dielectric layer of thickness, d, is deposited on a perfectly conducting substrate as shown below



If electromagnetic energy whose electric field is perpendicular to the paper is incident at an angle θ with respect to the normal as shown, obtain the following:

   a. Reflection coefficient
   b. The condition for surface waves to exist at the air dielectric interface

4. In a rectangular resonator obtain the equations which would yield the allowable resonant frequencies for the TM modes in the resonator.

5. Qualitatively describe the applications of a resonator.

ECE 550

First Examination

March 7, 2002

1. In a crystalline material which is a perfect dielectric the dielectric constant is represented by the following tensor

$$[\varepsilon] = \begin{array}{c} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \end{array} \begin{bmatrix} \varepsilon_{11} & 0 & 0 \\ 0 & \varepsilon_{11} & 0 \\ 0 & 0 & \varepsilon_{33} \end{bmatrix}$$

If a magnetic field is applied in the <αβ●> direction obtain the resulting electric field. Assume $e^{i\omega t}$ time variation.

2. A wave is propagating in a homogeneous isotropic medium that has zero conductivity but ~~finite dielectric losses~~. If the medium is nonmagnetic obtain an expression for the propagation term.

3. (a) What is Brewster's Angle?

(b) Under what conditions does a Brewster's angle exist?

(c) Prove that the Brewster's angle, θ_i, is given as follows

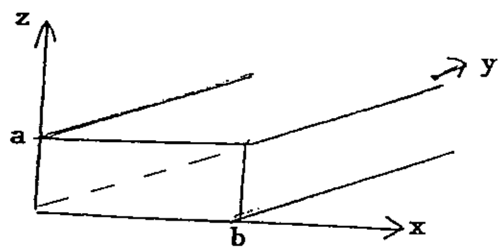
$$\theta_i = \tan^{-1}\sqrt{\frac{\varepsilon_2}{\varepsilon_1}}$$

where ε_1 and ε_2 are the dielectric constants of the two media of interest.

4. Under what conditions will a surface wave exist? Prove that the condition for the existence of a surface wave is given by

$$\theta_i > \sin^{-1} \sqrt{\frac{\mu_2 \varepsilon_2}{\mu_1 \varepsilon_1}}$$

5. For the rectangular waveguide structure shown below



obtain the cutoff frequency and electromagnetic fields for the TE_{on} mode. Assume that the walls are perfect conductors and the medium inside the waveguide is a perfect dielectric.

6. If the medium in the waveguide for problem 5 has losses due to finite conductivity, prove that the propagation term, γ, for this mode may be approximated as follows for frequencies, f, considerably larger than the cutoff frequency, fc, for this mode

$$\gamma \approx jk \sqrt{1 - (fc/f)^2}$$

when

$$k = \omega\sqrt{\mu\varepsilon}$$

ECE 550
First Exam
Closed Notes
March 6, 2003
Dr. John F. Vetelino

1. For the following media which are charge free derive the wave equation for the electric field. Define all terms.

   a. Perfect dielectric

   b. Medium with dielectric losses, constant permeability and no conductivity

   c. Medium with no conductivity, constant permeability and the dielectric constant is a function of position.

2. In a conducting medium obtain the following

   a. Group velocity

   b. Propagation and attenuation term

3. A wave travels in the < 1 1 0 > direction in a perfect dielectric medium and the electric field is in the positive z direction. Obtain an expression for the Poynting vector associated with this wave.

4. Explain in detail the conditions necessary for the existence of the following waves when an electromagnetic wave is incident on a medium

   a. Surface wave

   b. no reflected wave

5. In a lossless transmission line prove that the characteristic impedance is given as follows

$$z_o = \eta \frac{L}{\mu}$$

where

L = inductance per unit length
μ = permeability

and

η = resistance

2003 First Exam Answers

ECE550
3/25/03

I ① Perfect Dielectric

① $\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$ $\qquad \vec{J} = \sigma \vec{E}$ ⑥

② $\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$ $\qquad \vec{D} = \epsilon \vec{E}$ ⑤

③ $\nabla \cdot \vec{D} = 0$ $\qquad \vec{B} = \mu \vec{H}$ ⑦

④ $\nabla \cdot \vec{B} = 0$

$\sigma = 0$ take $\nabla\times$ of ①, substitute from ② using ⑦

$$\nabla(\nabla \cdot \vec{E}) - \nabla^2 \vec{E} - \mu\sigma \frac{\partial \vec{E}}{\partial t} - \mu \frac{\partial^2 \vec{E}}{\partial t^2}$$

ⓐ $$\boxed{\nabla^2 \vec{E} = -\mu\epsilon \frac{\partial^2}{\partial t^2}}$$

② σ is finite

ⓑ $$\boxed{\nabla^2 \vec{E} - \mu\sigma \frac{\partial \vec{E}}{\partial t} - \mu\epsilon \frac{\partial^2 \vec{E}}{\partial t^2} = 0}$$

ⓒ $\sigma = 0$ $\quad \mu =$ constant $\quad \epsilon = f(\text{position})$

$\underbrace{\nabla \cdot \epsilon \vec{E} = 0}_{\text{vector identity}} \Rightarrow \vec{E} \cdot \nabla \epsilon + \epsilon \nabla \cdot \vec{E} = 0$

$$\nabla \cdot \vec{E} = \frac{-\vec{E} \cdot \nabla \epsilon}{\epsilon}$$

$$\nabla\left(\frac{-\vec{E} \cdot \nabla \epsilon}{\epsilon}\right) - \nabla^2 \vec{E} = -\mu \epsilon \frac{\partial^2 \vec{E}}{\partial t^2}$$

$$\nabla^2 \vec{E} + \nabla\left(\frac{\vec{E}\,\nabla\epsilon}{\epsilon}\right) - \mu\epsilon \frac{\partial^2 \vec{E}}{\partial t^2}$$

2 Conducting Media

$\sigma =$ finite

assume $e^{j\omega t}$

$$\nabla^2 \vec{E} - \mu\sigma (j\omega)\vec{E} - \mu\epsilon(-\omega^2)\vec{E} = 0$$

$$\nabla^2 \vec{E} + \mu\omega\,\cancel{\mu\omega}(\epsilon\omega - j\sigma)\vec{E} = 0$$

in conducting media $\sigma \gg \epsilon\omega$

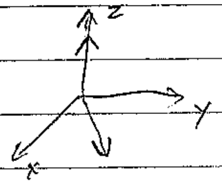
$$\nabla^2 \vec{E} - \gamma^2 \vec{E} = 0 \qquad \gamma^2 = j\omega\mu\sigma$$

$$\gamma = (1+i)\sqrt{\frac{\omega\mu\sigma}{2}}$$

$$E_z = E_{oz}\, e^{\underbrace{-i\sqrt{\frac{\omega\mu\sigma}{2}}x}_{\substack{\text{Propagation}\\ \beta}} \underbrace{-\sqrt{\frac{\omega\mu\sigma}{2}}x}_{\substack{\text{attenuation}\\ \alpha}}}$$

group velocity $d\omega/d\beta \Rightarrow \quad \beta = \sqrt{\frac{\omega\mu\sigma}{2}}$

(3)



$$\vec{E} = \vec{K} E_z$$

$$\vec{S} = \vec{E} \times \vec{H} \qquad e^{j\omega t} \text{ time variation}$$

$$E_z = e^{-j(\vec{k}\cdot\vec{r}) + j\omega t}$$

$$\vec{r} = \vec{i}x + \vec{j}y$$

$$k = \vec{i}k_x + \vec{j}k_y$$

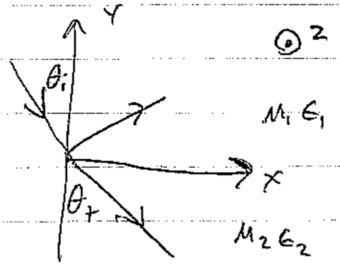
$$E_z = E_{0z} e^{-j\frac{k}{\sqrt{2}}(x+y) + j\omega t} \qquad k_x = k_y = k/\sqrt{2}$$

$$\nabla \times \vec{E} = -j\omega\mu\vec{H} \qquad \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 0 & 0 & E_z \end{vmatrix} = -j\omega\mu[\vec{i}H_x + \vec{j}H_y]$$

$$H_x = -\frac{1}{j\omega\mu}\,\partial E_z/\partial y$$

$$H_y = \frac{1}{j\omega\mu}\,\partial E_z/\partial x$$

(4)



assume media non conducting

surface wave propagation in x-direction, attenuation in y-direction

Snell's Law

$$\frac{\sin\theta_t}{\sin\theta_i} = \sqrt{\frac{\mu_1 \epsilon_1}{\mu_2 \epsilon_2}}$$

$$\sin\theta_t = \sqrt{\frac{\mu_1 \epsilon_1}{\mu_2 \epsilon_2}}\ \sin\theta_i$$

if $\sqrt{\frac{\mu_1 \epsilon_1}{\mu_2 \epsilon_2}} > 1$, $\sin\theta_t$ may be >1

for $\theta_t = 90^\circ$ $\Rightarrow$ $\sin\theta_i = \sqrt{\frac{\mu_1 \epsilon_1}{\mu_2 \epsilon_2}}$, $\mu_1 \simeq \mu_2$

$\boxed{\epsilon_1 > \epsilon_2}$

$$\theta_i \geq \sin^{-1}\sqrt{\frac{\mu_2 \epsilon_2}{\mu_1 \epsilon_1}}$$

$\boxed{\theta_{critical} \geq \sin^{-1}\sqrt{\frac{\mu_2 \epsilon_2}{\mu_1 \epsilon_1}} \quad \text{surface waves}}$

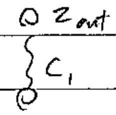
no reflected wave $|\Gamma| = 0$

$\vec{H}$ must be $\parallel$ to interface

$$n_2 \cos\theta_t = n_1 \cos\theta_i$$

$$\boxed{\theta_i = \tan^{-1}\sqrt{\frac{\epsilon_2}{\epsilon_1}}}$$

(5) $V = \int_{C_1} \vec{E}\cdot\vec{dl}$



$E = \eta \vec{H} \times \vec{u}_z$ ← normal to lines

$V = \eta \int_{C_1} \vec{H}\times\vec{u}_z \cdot \vec{dl} = \eta \int_{C_1} H_n \, dl$

$L = \Psi/I = \int_{C_1} \frac{\vec{B}\cdot\vec{dl}}{I} = \frac{\mu}{I}\int_{C_1} H_n \, dl = V/\eta$

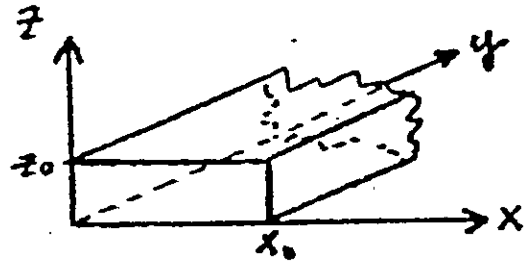
$L = \frac{\mu V}{\eta I} \Rightarrow \boxed{\frac{V}{I} = \frac{\eta L}{\mu}}$

Do problems 2 & 3 of exam (part 2)
also have field representations

Name [redacted]

ECE 550
FINAL EXAM
Open Notes and Books
May 5, 1998

1. For the air filled waveguide shown below set up the general expression for the $\vec{E}$ and $\vec{H}$ fields and the propagation constant for the TM_{mn} modes.



all walls perfectly conducting

If $z_o > x_o$ what is the dominant mode? If the waveguide is cut at $y = y_o$ and a perfectly conducting plate put in, derive the expression for the resonant frequencies of the resulting structure.

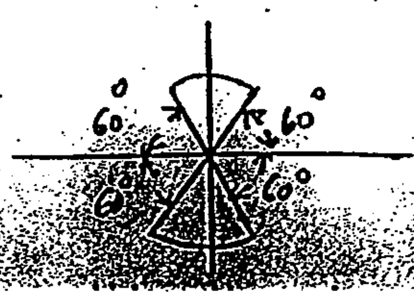
2. A centerfed linear antenna is $(7/2)\lambda$ in length. Calculate and plot the far field. Same as 1

3. If the linear antenna described in Problem 2 is configured as a traveling wave antenna, obtain an expression for the far field. same as 1

4. For a 16 element end fire array with an inter-element spacing of $\lambda/4$, obtain the following Show steps to obtain for the final
   a. approximate field pattern.
   b. width of the principal lobe.

5. If the 16 element array in Problem 4 is changed to a broadside array and then placed a distance $\lambda/4$ over a perfectly conducting plane, obtain an expression for the far field pattern.

6. A desired radiation pattern is shown below.



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FINAL EXAMINATION
ECE 550 – Electromagnetics
Dr. John F. Vetelino
May 7, 1999
Open Notes

1. A short dipole antenna of length, L, is energized at the center by the following current excitation

$$I(z) = I_m \sin\left[k\left(\frac{L}{2} - |z|\right)\right]$$

pg 220-221 Kraus ch 5

where k = wave number

If the dipole is (7/2)λ long obtain an expression for the horizontal and vertical far field patterns and sketch each one.

2. Prove that the radiation resistance of a short dipole antenna may be expressed as follows:

$$R_{rad} = 30\{S_1(b) - [S_1(2b) - S_1(b)]\cos b +$$

$$[Si(2b) - Si(b)]\sin b + [1 + \cos b]S_1(b) - Si(b)\sin b\}$$

where

$$b = kL$$

$$Si(x) = \int_o^x \frac{\sin v}{v} dv$$

$$Ci(x) = -\int_x^\infty \frac{\cos v}{v} dv$$

and

$$S_1(x) = \ln x - Ci(x) + C$$

3. In a traveling wave antenna the current distribution is given as follows

$$I(z) = I_m e^{\cancel{j}kz} \qquad I_m e^{-jkz}$$

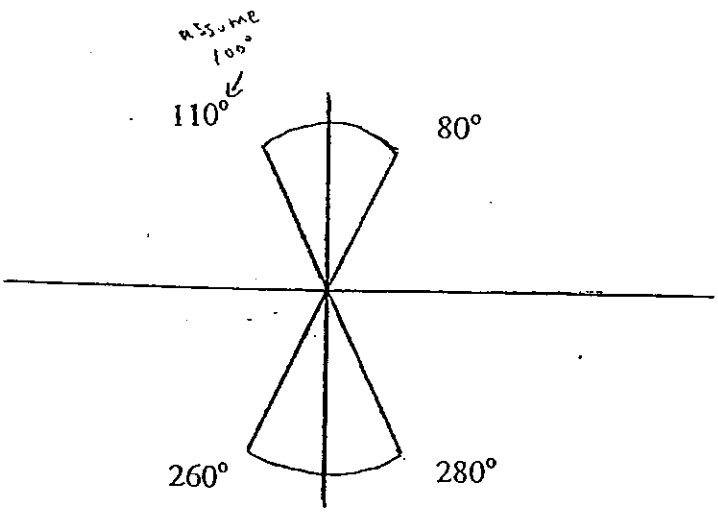
end of CH 5
Kraus

If the antenna is two wavelengths long, obtain an expression for the radiation pattern and sketch the pattern.

4. In an end fire array obtain an expression for the width of the principal maximum.

5. A broadside array of 16 elements is fed in phase. Carefully outline the necessary steps to obtain the radiation pattern.

6. Obtain the antenna array which will give you the following pattern.



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ECE 550
Examination (Open Notes)
Dr. J. F. Vetelino
May 17, 2000

1. For a linear center fed dipole antenna answer the following questions.

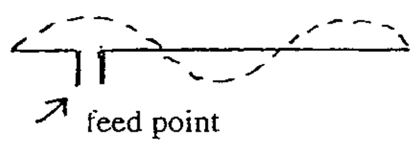
   a. Develop a general expression which is valid for all values of $\underline{r}$ for the electric and magnetic fields radiating from this antenna if the current distribution on the antenna is given by the following relation.

$$I(z) = I_m \sin\left[k\left(\frac{L}{2} - |z|\right)\right]$$

   b. Plot the horizontal and vertical radiation patterns for this antenna in the far field if $L = 4\lambda$.

   c. Develop an expression for the radiation resistance of this antenna.

2. For a linear dipole antenna whose current distribution is given as follows

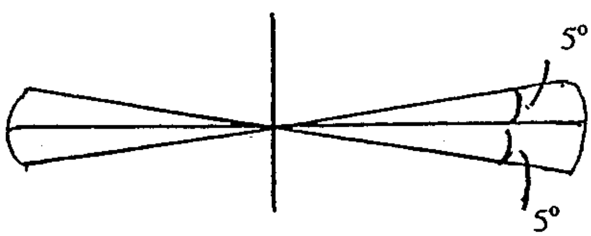


dotted line indicates current distribution

calculate the far field radiation pattern.

3. For a 24 element end fire array calculate the far field radiation pattern if the element spacing is $\lambda/2$.

4. The following far field radiation pattern is desired.



Assuming that you have 21 elements, obtain the antenna configuration which would best approximate the desired pattern.

5. Design an 8 element array having a spacing of $d = \lambda/4$ between elements and side lobe level down 19.1 db.

ECE 550
Electromagnetic Theory
Final Exam
May 15, 2001
Dr. John F. Vetelino
Open Notes and Book

$R_{rad} = \frac{\eta}{2\pi}\int_0^{\pi}\frac{\left[\cos\left(k\frac{L}{2}\cos\theta\right)-\cos\left(k\frac{L}{2}\right)\right]^2}{\sin\theta}d\theta$

1. A center fed short dipole antenna is four wavelengths long. Obtain and plot the horizontal and vertical radiation patterns from this antenna. Obtain an equation for the radiation resistance of this antenna. $E_\theta = \frac{60 I_m}{r}\left[\frac{\cos\left(\frac{kL}{2}\cos\theta\right)-\cos\left(k\frac{L}{2}\right)}{\sin\theta}\right]$

always circle $\theta = 90^\circ$

2. Assume that you have a uniform linear endfire antenna array. Prove that the width of the principal maxima is the following.

$$2\left(\frac{2\lambda}{nd}\right)^{1/2}$$

where λ = wavelength

n = number of elements in the array

and

d = element spacing.

3. Assume that ~~the array described in problem 2 has 20~~ elements and is operated as a broadside array. If the element spacing is $\lambda/4$ obtain the radiation pattern.

$n=20$, $d=\frac{\lambda}{4}$, $E=\left|\frac{\sin n\frac{\psi}{2}}{\sin\frac{\psi}{2}}\right|$, $\alpha=0$, $\psi = kd\cos\phi+\alpha$, $kd = \frac{2\pi}{\lambda}\frac{\lambda}{4}=\frac{\pi}{2}$, $\psi=\frac{\pi}{2}\cos\phi$

$E = \frac{\sin 20\left(\frac{\frac{\pi}{2}\cos\phi}{2}\right)}{\sin\left(\frac{\frac{\pi}{2}\cos\phi}{2}\right)} = \left|\frac{\sin 5\pi\cos\phi}{\sin\frac{\pi}{4}\cos\phi}\right|$

4. Obtain the broadside radiation pattern of a 16 element array spaced $\lambda/2$ apart.

$\alpha=0$, $d=\frac{\lambda}{2}$, $kd = \frac{2\pi}{\lambda}\frac{\lambda}{2}=\pi$, $\psi=\pi\cos\phi$, $E=\left|\frac{\sin 16\left(\frac{\pi\cos\phi}{2}\right)}{\sin\frac{\pi\cos\phi}{2}}\right| \Rightarrow \left|\frac{\sin 8\pi\cos\phi}{\sin\frac{\pi}{2}\cos\phi}\right| = E$

5. Design a 12 element endfire array having an element spacing of $\lambda/2$ and a side lobe level 40 dbs down. What is the width of principal maxima.

$n=12$, $\alpha=-kd$, $d=\frac{\lambda}{2}$, $P.M. = 2\left(\frac{2\lambda}{nd}\right)^{1/2}$, $2\left(\frac{2\cdot\lambda}{\frac{n\lambda}{2}}\right)$, $2\left(\frac{4}{n}\right)$

$= 2\left(\frac{4}{n}\right)^{1/2} = 2\left(\frac{4}{12}\right)^{1/2}$
1.154

lect. 20 pg 5

$|E| = |1+z+z^2\cdots z^{n-1}|$

$\left|\frac{E_T}{E_0}\right| = \left|\frac{\sin n\frac{\psi}{2}}{\sin\frac{\psi}{2}}\right|$

P.M $\Rightarrow \psi = 0$ $\quad E_T/E_0 \Rightarrow n$

S.M. $\frac{n\psi}{2} = \pm(2p+1)\frac{\pi}{2}$ $p=1,2,3\ldots$

First Second Max. is 13.5 db below the P.M.

$|E| = |1+z+z^2\cdots z^{n-1}|^2$

27db below P.M.

$|E| = |1+z+z^2\cdots z^{n-1}|^3$

First SM: $P=1$

$\frac{\psi}{2} = \pm\frac{3\pi}{2n}$

$\left|\frac{n\sin\frac{\psi}{2}}{\sin\psi/2}\right|^3 = \left(\frac{2n}{3\pi}\right)^3 = 2.0197$

$-20\log 2.0197$

[redacted]

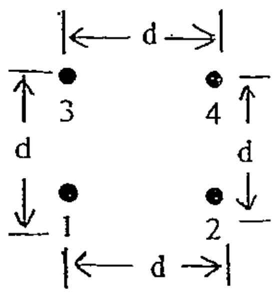
Final Examination
ECE 550
Dr. John F. Vetelino
May 9, 2002

1. In a short dipole antenna derive expressions for the electric and magnetic fields in the near field.

2. For the following square 4 element short dipole antenna array the current in the ith element is given as

$$I_i = k_i \angle \alpha_i \quad i = 1, 2, 3, 4$$

obtain an expression for the far field radiation pattern



3. For the uniform linear end fire array with n elements and all current magnitudes the same prove the following

a. $|E_T/E_0| = n$ for the principal maxima

where $E_T =$ total radiation field

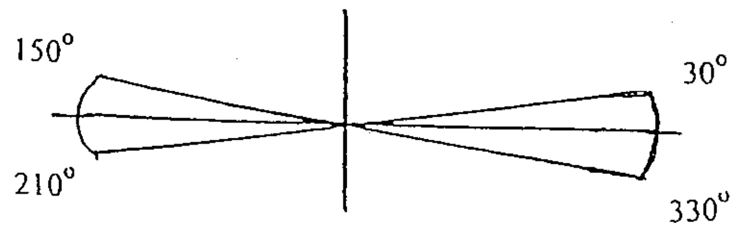
and $E_0 =$ radiation field from element zero

b. Ratio of the secondary maxima to the primary maxima is .212.

c. Width of the principal lobe for an element spacing of 2λ is $2/(n)^{1/2}$.

4. In a particular application a single narrow main lobe with no side lobes is desired. If there are 15 elements in the array, draw the geometric configuration of the array. Indicate the spacing, current values and also draw the radiation pattern.

5. Design an antenna array which will produce the following radiation pattern.



6. Design a 7 element broadside array having a spacing of $\lambda/2$ between elements and a side lobe 30db down from the main lobe. Draw the pattern for this array.

ECE 550 Exam
Electromagnetics
Closed Notes and Books
May 15, 2003
John F. Vetelino

1. The standard equation which is used in the study of antennas is the Hemholtz equation.

   a. Derive this equation.
   b. Obtain the relationships between the magnetic and electric fields and the magnetic vector potential.

2. For the small electric dipole obtain a relationship for the outward directed complex power.

$\bar{P}_f = S = E \times H = \frac{E E^*}{\eta}$

3. Describe in detail the following two antenna theorems.

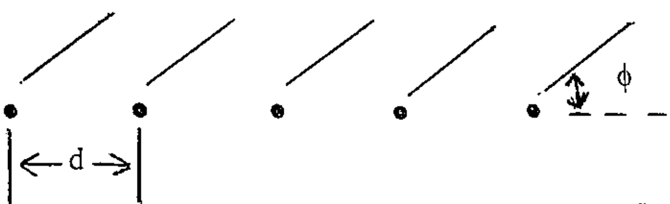
   a. Maximum Power Transfer Theorem
   b. Reciprocity Theorem

4. In a linear uniform antenna array obtain the relationships between the angle, ϕ, shown below, the progressive phase shift and the spacing between elements for the following

   a. principle maxima of the radiation pattern

   b. pattern nulls of the radiation pattern.

$A = \vec{H} + j\vec{E}$



$\nabla \times A =$

$\nabla \times \nabla \times A = \nabla(\nabla \cdot A) - \nabla^2 A$

$\nabla^2 A + k^2 A = -J$

$\nabla \times E = k\vec{B}$

$\nabla \times H = k\vec{D}$

$\nabla \cdot B = \cancel{0}\, kH$

$\nabla \cdot D = 0\ kE$

$\nabla \times B = J$

$E \cdot H^* = A$

$E = \frac{H}{K}$

∇

$-J = \nabla \times B$

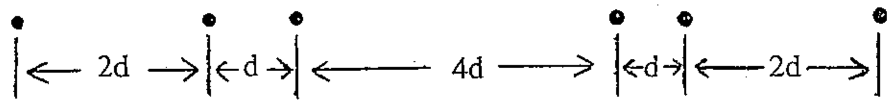
$\nabla^2 \vec{A} = J$

$\nabla \times B = J$

ECE 550 Exam
Electromagnetics
Open Notes
May 15, 2003
Dr. John F. Vetelino

1. A uniform broadside array of 16 elements with a interelement spacing of $\lambda/8$ is located a short distance above the earth. Obtain the resulting far field radiation pattern.

2. For the following array



assume the following

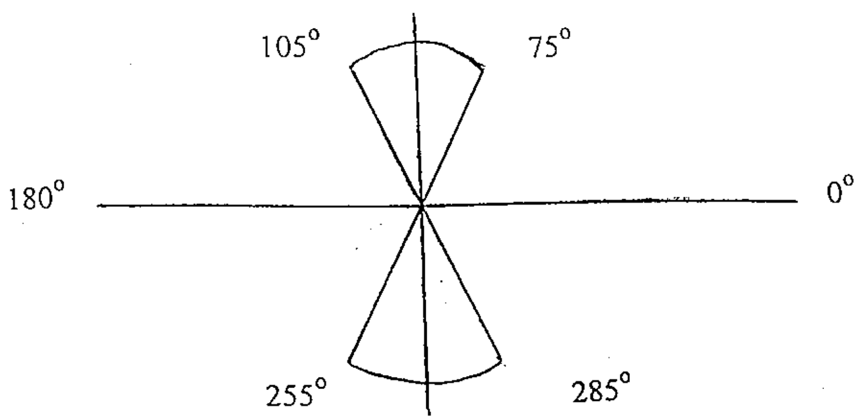
(1) The shortest spacing between elements is d.

(2) The currents in each element are different.

(3) The progressive phase shift lead is zero.

Develop a relation which if plotted would give you the antenna pattern.

3. For the following antenna pattern obtain an antenna configuration which will approximately yield this pattern.



4. Design a 8 element end fire array having a side lobe level down 40 db and an element spacing of $d = \lambda/2$

$$(\sqrt{3}-1)\sqrt{2} = \frac{\sqrt{8}-\sqrt{2}}{4}$$