

## General Definitions

$$X(z) = \sum_{n=-\infty}^{\infty} x(n)z^{-n}$$

$$x(n) = \frac{1}{2\pi j} \oint X(z)z^{n-1}dz$$

$$X^+(z) = \sum_{n=0}^{\infty} x(n)z^{-n}$$

$$x(n) = \frac{1}{2\pi j} \oint X^+(z)z^{n-1}dz, \quad n \geq 0$$

$$X(\omega) = \sum_{n=-\infty}^{\infty} x(n)e^{-j\omega n}$$

$$x(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(\omega)e^{j\omega n}d\omega$$

$$X(k) = \sum_{n=0}^{N-1} x(n)e^{-j2\pi kn/N}$$

$$x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k)e^{j2\pi kn/N}$$

## Linear Systems

$$y(n) = x(n) * h(n) = \sum_{k=-\infty}^{\infty} x(k)h(n-k) = \sum_{k=-\infty}^{\infty} h(k)x(n-k)$$

$$Y(z) = X(z)H(z) \quad Y(\omega) = X(\omega)H(\omega)$$

## Transform Properties

— Double-Sided Z-Transforms —

$$a_1x_1(n) + a_2x_2(n) \Leftrightarrow a_1X_1(z) + a_2X_2(z)$$

$$x(n-k) \Leftrightarrow z^{-k}X(z)$$

$$a^n x(n) \Leftrightarrow X(z/a)$$

$$x(n) * h(n) \Leftrightarrow X(z)H(z)$$

— Single-Sided Z-Transforms —

$$x(n-k) \Leftrightarrow z^{-k}X^+(z) + x(-k) + x(-k+1)z^{-1} + \dots + x(-1)z^{-k+1}$$

$$x(n+k) \Leftrightarrow z^kX^+(z) - x(k-1)z - x(k-2)z^2 - \dots - x(0)z^k$$

— DTFT —

$$e^{j\omega_0 n}x(n) \Leftrightarrow X(\omega - \omega_0)$$

$$x_1(n)x_2(n) \Leftrightarrow \frac{1}{2\pi} \int_{-\pi}^{\pi} X_1(\lambda)X_2(\omega - \lambda)d\lambda$$

## Transform Pairs

— Double-Sided Z-Transforms —

$$\delta(n) \Leftrightarrow 1$$

$$a^n u(n) \Leftrightarrow \frac{z}{z-a} = \frac{1}{1-az^{-1}}, \quad |z| > |a|$$

$$-a^n u(-n-1) \Leftrightarrow \frac{z}{z-a} = \frac{1}{1-az^{-1}}, \quad |z| < |a|$$

$$a^n \cos(\omega_0 n)u(n) \Leftrightarrow \frac{1-az^{-1}\cos(\omega_0)}{1-2az^{-1}\cos(\omega_0)+a^2z^{-2}}, \quad |z| > |a|$$

$$a^n \sin(\omega_0 n)u(n) \Leftrightarrow \frac{az^{-1}\sin(\omega_0)}{1-2az^{-1}\cos(\omega_0)+a^2z^{-2}}, \quad |z| > |a|$$

— DTFT (“in the limit”) —

$$u(n) \Leftrightarrow \frac{e^{j\omega}}{e^{j\omega}-1} + \sum_{k=-\infty}^{\infty} \pi\delta(\omega-2\pi k)$$

$$e^{j\omega_0 n} \Leftrightarrow \sum_{k=-\infty}^{\infty} 2\pi\delta(\omega-\omega_0-2\pi k)$$

## Sampling Theorem

$$X(F/F_s) = F_s \sum_{k=-\infty}^{\infty} X_a(F-kF_s)$$

$$f = F/F_s + k \quad X(f) = F_s \sum_{k=-\infty}^{\infty} X_a(F_s(f-k))$$

$$x_a(t) = \sum_{n=-\infty}^{\infty} x(n) \frac{\sin(\pi(t-nT_s)/T_s)}{\pi(t-nT_s)/T_s}$$

## Bilinear Transformations

$$s = \frac{z-1}{z+1} \quad z = \frac{s+1}{s-1}$$

$$\Omega = \tan(\omega/2)$$

## Power Series

$$\sum_{n=0}^{\infty} x^n = \frac{1}{1-x} \quad |x| < 1$$

$$\sum_{n=0}^{\infty} \frac{x^n}{n!} = e^x \quad |x| < \infty$$

## Trigonometry

(Standard Trig Identities will be furnished on request)

$\theta$	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$
$\sin(\theta)$	0	1/2	$1/\sqrt{2}$	$\sqrt{3}/2$	1
$\cos(\theta)$	1	$\sqrt{3}/2$	$1/\sqrt{2}$	1/2	0
$\tan(\theta)$	0	$1/\sqrt{3}$	1	$\sqrt{3}$	$\infty$