General Definitions

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$$

$$x(n) = \frac{1}{2\pi j} \oint X(z) z^{n-1} dz$$

$$X^+(z) = \sum_{n=0}^{\infty} x(n) z^{-n}$$

$$x(n) = \frac{1}{2\pi j} \oint X^+(z) z^{n-1} dz, \ n \geq 0$$

$$X(\omega) = \sum_{n=-\infty}^{\infty} x(n) e^{-j\omega n}$$

$$x(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(\omega) e^{j\omega n} d\omega$$

$$X(k) = \sum_{n=0}^{N-1} x(n) e^{-j2\pi kn/N}$$

$$x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k) e^{j2\pi kn/N}$$

Linear Systems

$$y(n) = x(n) * h(n) = \sum_{k=-\infty}^{\infty} x(k) h(n-k) = \sum_{k=-\infty}^{\infty} h(k) x(n-k)$$

$$Y(z) = X(z) H(z) \qquad Y(\omega) = X(\omega) H(\omega)$$

Transform Properties

— Double-Sided Z-Transforms —

$$a_1 x_1(n) + a_2 x_2(n) \rightleftharpoons a_1 X_1(z) + a_2 X_2(z)$$

$$x(n-k) \rightleftharpoons z^{-k} X(z)$$

$$a^n x(n) \rightleftharpoons X(z/a)$$

$$x(n) * h(n) \rightleftharpoons X(z) H(z)$$

— Single-Sided Z-Transforms —

$$x(n-k) \rightleftharpoons z^{-k} X^+(z) + x(-k) + x(-k+1) z^{-1} + \ldots + x(-1) z^{-k+1}$$

$$x(n+k) \rightleftharpoons z^k X^+(z) - x(k-1) z - x(k-2) z^2 - \ldots - x(0) z^k$$

— DTFT —

$$e^{j\omega_0 n} x(n) \rightleftharpoons X(\omega - \omega_0)$$

$$x_1(n) x_2(n) \rightleftharpoons \frac{1}{2\pi} \int_{-\pi}^{\pi} X_1(\lambda) X_2(\omega - \lambda) d\lambda$$

Transform Pairs

— Double-Sided Z-Transforms —

$$\delta(n) \rightleftharpoons 1$$

$$a^n u(n) \rightleftharpoons \frac{z}{z-a} = \frac{1}{1-az^{-1}}, \quad |z| > |a|$$

$$-a^n u(-n-1) \rightleftharpoons \frac{z}{z-a} = \frac{1}{1-az^{-1}}, \quad |z| < |a|$$

$$a^n \cos(\omega_0 n) u(n) \rightleftharpoons \frac{1 - az^{-1}\cos(\omega_0)}{1 - 2az^{-1}\cos(\omega_0) + a^2 z^{-2}}, \quad |z| > |a|$$

$$a^n \sin(\omega_0 n) u(n) \rightleftharpoons \frac{az^{-1}\sin(\omega_0)}{1 - 2az^{-1}\cos(\omega_0) + a^2 z^{-2}}, \quad |z| > |a|$$

— DTFT ("in the limit") —

$$u(n) \rightleftharpoons \frac{e^{j\omega}}{e^{j\omega} - 1} + \sum_{k=-\infty}^{\infty} \pi \delta(\omega - 2\pi k)$$

$$e^{j\omega_0 n} \rightleftharpoons \sum_{k=-\infty}^{\infty} 2\pi \delta(\omega - \omega_0 - 2\pi k)$$

Sampling Theorem

$$X(F/F_s) = F_s \sum_{k=-\infty}^{\infty} X_c(F - kF_s)$$

$$f = F/F_s + k \qquad X(f) = F_s \sum_{k=-\infty}^{\infty} X_c(F_s(f-k))$$

$$x_c(t) = \sum_{n=-\infty}^{\infty} x(n) \frac{\sin(\pi(t - nT_s)/T_s)}{\pi(t - nT_s)/T_s}$$

Power Series

$$\sum_{n=0}^{\infty} x^n = \frac{1}{1-x} \quad |x| < 1$$

$$\sum_{n=0}^{\infty} \frac{x^n}{n!} = e^x \quad |x| < \infty$$

Trigonometry

(Standard Trig Identities will be furnished on request)

θ	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$
$\sin(\theta)$	0	$1/2$	$1/\sqrt{2}$	$\sqrt{3}/2$	1
$\cos(\theta)$	1	$\sqrt{3}/2$	$1/\sqrt{2}$	$1/2$	0
$\tan(\theta)$	0	$1/\sqrt{3}$	1	$\sqrt{3}$	∞