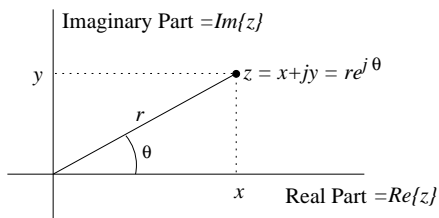


# Things You Should Know

(and be able to do without computers/calculators)

## Complex Numbers



$$z = x + jy = r \cos(\theta) + jr \sin(\theta)$$

$$|z| = r = \sqrt{x^2 + y^2} \quad \angle z = \theta = \tan^{-1}\left(\frac{y}{x}\right)$$

$$e^{j\theta} = \cos(\theta) + j \sin(\theta) \quad |e^{j\theta}| = 1$$

$$\left| \frac{z_1}{z_2} \right| = \frac{|z_1|}{|z_2|} \quad |z_1 z_2| = |z_1| |z_2| \quad |z_1 + z_2| \leq |z_1| + |z_2|$$

$$\frac{z_1}{z_2} = \frac{r_1 e^{j\theta_1}}{r_2 e^{j\theta_2}} = \frac{r_1}{r_2} e^{j(\theta_1 - \theta_2)}$$

$$z_1 z_2 = (r_1 e^{j\theta_1})(r_2 e^{j\theta_2}) = r_1 r_2 e^{j(\theta_1 + \theta_2)}$$

## Algebra

$$\frac{e^x}{e^y} = e^{x-y} \quad e^x e^y = e^{x+y}$$

$$\log\left(\frac{xy}{z}\right) = \log(x) + \log(y) - \log(z)$$

## Calculus

$$\frac{d}{dx}(f(x) + g(x)) = \frac{df}{dx} + \frac{dg}{dx}$$

$$\frac{d}{dx}(f(x)g(x)) = f(x)\frac{dg}{dx} + g(x)\frac{df}{dx}$$

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{g(x)\frac{df}{dx} - f(x)\frac{dg}{dx}}{g^2(x)}$$

$$\frac{d}{dx}(f(g(x))) = \left(\frac{df}{dg}\right)\left(\frac{dg}{dx}\right)$$

## Common Integrals/Derivatives

$$\frac{d}{dx}e^{ax} = ae^{ax} \quad \int e^{ax} dx = \frac{1}{a}e^{ax} + c$$

$$\frac{d}{dx}x^n = nx^{n-1} \quad \int x^n dx = \frac{1}{n+1}x^{n+1} + c \quad (n \neq -1)$$

$$\int \frac{1}{x} dx = \ln(x) + c$$

## Examples

$$e^{j\pi/2} = j, \quad e^{j\pi} = -1, \quad e^{j3\pi/2} = -j, \quad e^{j2\pi} = 1$$

$$1 + j = \sqrt{2}e^{j\pi/4}, \quad 1 - j = \sqrt{2}e^{-j\pi/4}$$

$$|3 + j5|^2 = 34$$

$$(3 + j5)^2 = -16 + j30$$

$$|2e^{j\pi/3}|^2 = 4$$

$$\frac{2e^{j\pi/3}}{3e^{-j\pi/6}} = \frac{2}{3}j$$

$$\left| \frac{1 + j2}{3e^{-j\pi/6}} \right|^2 = \frac{5}{9}$$

$$\frac{e^{-j\pi}}{j} = j$$

$$(2e^{j\pi/7})^3 = 8e^{j3\pi/7}$$

$$\left[ \frac{1}{\sqrt{2}}(1 + j) \right]^{100} = -1$$

$$|1 + j5 + 5\sqrt{2}e^{j3\pi/4}|^2 = 116$$

$$\int_0^T e^{j2\pi nt/T} dt = \begin{cases} 0 & n \neq 0 \\ T & n = 0 \end{cases}$$

$$\operatorname{Re}\{5e^{j\pi/7} e^{j\omega_0 t}\} = 5 \cos(\omega_c t + \pi/7)$$

$$\operatorname{Re}\{3e^{-5t+j\pi/3} e^{j200\pi t}\} = 3e^{-5t} \cos(200\pi t + \pi/3)$$

$$\frac{d}{dt}(te^{at}) = (at + 1)e^{at}$$

$$\int_0^T e^{-3t} e^{2t} dt = 1 - e^{-T}$$

$$\int_{-2}^1 e^{-t/10} dt = 10(e^{0.2} - e^{-0.1})$$

$$\int_1^2 x^3 dx = \frac{15}{4}$$