ECE-486 Homework 7, Spring 2009

1. Let $X(k)$, $0 \leq k \leq N-1$ be the N-point DFT of a length N sequence $x(n)$, with N even. Determine the N-point DFTs of the following length-N sequences in terms of $X(k)$.

   (a) $w(n) = x((n-m_1)_N) + x((n-m_2)_N)$

   (b)
   $$g(n) = \begin{cases} x(n) & n \text{ even} \\ 0 & n \text{ odd} \end{cases}$$

   (c) $x(n) \textcircled{N} x(n)$

2. A signal processor operates with a sample frequency of $F_s = 48$ ksps. A table look-up (similar to that used in lab 4) is to be used to generate values of a cosine function at a frequency within 5 Hz of 12.3456 kHz.

   (a) Determine the length of the smallest table of cosine values which can meet this requirement. Give an equation for the n^{th} entry of the table, and indicate the exact frequency which is generated by periodically accessing the table at the 48 ksps rate. Describe how you found your solution.

   (b) Repeat part 2a assuming a 1 Hz tolerance (rather than 5 Hz) is used.

   (c) Repeat part 2a assuming a 0.1 Hz tolerance (rather than 5 Hz) is used.

   (d) An alternative to using a table look-up is provided by a "numerical oscillator". Numerical Oscillators are described in the text, and on problem 4 of the 2004 ECE486 Test 1 (on the web site). The idea is to use a 2^{nd}-order system with poles on the unit-circle so that the zero-input response of the system will oscillate at a desired frequency.

   For the above frequency, give the difference equation for the numerical oscillator which will generate the cosine. Give numerical values for all filter coefficients, and indicate the required filter initial conditions.