ECE-486 Homework 6, Spring 2009

1. Define $x(n) = A\cos(2\pi f_0 n)$

   (a) Write down and plot the DTFT of $x(n)$.

   (b) Write down the N-point DFT of $x(n)$ (No credit for just copying the generic DFT formula... Give the transform result for this $x(n)$). Give your equations in terms of the parameters f_0, A, and N.

   (c) Explain how your results would change if the sequence $x(n)$ were multiplied by an N-point window function $w(n)$ prior to calculation of the DFT. How would the height of the major peaks in the DFT change?

2. The course www site provides a link to a data file containing 256 samples of a distorted periodic signal, samples using a sampling frequency of 48 ksps. Assume that the data provided is in units of volts.

   The link to the data file is:
   http://www.eece.maine.edu/˜hummels/classes/ece486/data/hw6data.txt

   (a) Identify frequencies corresponding to the dominant tones of the signal.

   (b) Find the amplitude (in volts) of the largest tone present. Describe how you obtained your answer.

3. (This is a 2005 Test 2 question: You should be able to work this problem by hand... but please check your answer using MATLAB.) You may use the following transform pair

$$r(n) = \begin{cases} 1 & n = 0, \ldots, M-1 \\ 0 & \text{elsewhere} \end{cases} \qquad R(f) = \frac{\sin(M\pi f)}{\sin(\pi f)} \exp\left(-j\pi(M-1)f\right)$$

   Let $r(n)$ be defined as above as a width $M = 12$ rectangular pulse, and let $R(k)$, $k = 0, 1, \ldots, 15$ be the 16-point DFT of $r(n)$.

   (a) Give the expression for $R(k)$ (closed-form — no summations or integrals).

   (b) Define $Y(k) = R^2(k)$, $k = 0, 1, \ldots, 15$. Sketch the 16-point inverse-DFT of $Y(k)$.

   (c) Is a filter with impulse response $r(n)$ a linear phase filter? (Justify your answer.) If so, find the delay associated with the filter.

   (d) Define $Y(k) = R(k)e^{j2\pi\cdot 2\cdot k/N} + R(k)e^{-j2\pi k/N}$. Sketch the inverse 16-point DFT of $Y(k)$.

   (e) Now let $x(n)$, $n = 0, 1, \ldots 127$ be a 128-sample discrete-time sequence. To evaluate the convolution of $x(n)$ and $r(n)$, 128-point DFTs are used to evaluate $X(k)$ and $R(k)$. An output sequence $y(n)$, $n = 0, 1, \ldots 127$ is then determined using the inverse DFT of the product $X(k)R(k)$. Identify the sample indexes for which $y(n)$ provides the *linear* convolution of $x(n)$ and $r(n)$.