

ECE-486 Homework 5, Spring 2009

1. Plot (using MATLAB, Mathcad, etc) the DTFT of

$$x(n) = \alpha^n \cos(2\pi(0.2)n) u(n)$$

for $\alpha = 0.1, 0.2, 0.3, \dots, 0.9$ (Plot all of the graphs on the same set of axes). Sketch the pole-zero diagram of $X(z)$, and label your sketch in terms of the parameter α .

2. Define $H(f)$ to be an *ideal* lowpass filter with bandwidth 0.1 cycles/sample:

$$H(f) = \begin{cases} 1 & |f| < 0.1 \\ 0 & 0.1 < |f| < 0.5 \end{cases}$$

- (a) Use the inverse DTFT to evaluate the impulse response $h(n)$ of this system. (Your result should be symmetric, since $H(f)$ is real. You should also find that this is an IIR non-causal system.)
(b) Now define an FIR filter based on your calculated $h(n)$ from part 2a.

$$h_N(n) = \begin{cases} h(n) & -N \leq n \leq N \\ 0 & \text{elsewhere.} \end{cases}$$

The (real) transfer function associated with this FIR filter is

$$H_N(f) = \sum_{n=-N}^N h_2(n) e^{-j2\pi f n}$$

Plot the magnitude response (showing the filter gain in dB) for $H_N(f)$ for $N = 10, 20, 50,$ and 100 . (Show all four curves on the same graph).

- (c) To create a *causal* FIR filter, the truncated impulse response $h_N(n)$ may be delayed by N samples. How will the magnitude responses for these filters differ from those of the non-causal filters of part 2b? Describe the phase responses of these filters.
3. A causal first order filter is described by

$$y(n) = ay(n-1) + bx(n)$$

The parameters a and b may be assumed to be real.

- (a) Find the impulse response of the system.
(b) Find the conditions on a and b which result in a stable system.
(c) Find the conditions on a and b which result in a filter with a DC gain of one.
(d) Find the conditions on a and b which result in a lowpass filter characteristic.
4. Consider the FIR filter

$$y(n) = x(n) - x(n-4)$$

- (a) Compute and sketch the magnitude and phase response of the filter.
(b) Compute the system output $y(n)$ for

$$x(n) = \cos\left(\frac{n\pi}{2}\right) + \cos\left(\frac{n\pi}{4}\right) \quad -\infty < n < \infty$$

- (c) Explain the results obtained in part 4b in terms of the answer given in part 4a.