ECE-486 Homework 4, Spring 2009

1. A causal system has poles at $z = 0$ and $z = -0.8$ and zeros at $z = 1.5e^{j\pi/6}$ and $z = 1.5e^{-j\pi/6}$.

   (a) Determine the transfer function $H(z)$ and impulse response of the system given that the system DC gain is 1.

   (b) Is this system stable?

   (c) Sketch a possible implementation of the system and determine the corresponding difference equation description.

2. Consider a causal system with

$$H(z) = \frac{z^{-1} + \frac{1}{2}z^{-2}}{1 - \frac{3}{5}z^{-1} + \frac{2}{25}z^{-2}}$$

   (a) Find the impulse response of the system.

   (b) Find the zero-state step response of the system.

   (c) Find the step response if $y(-1) = 1$ and $y(-2) = 2$.

3. We want to design a causal discrete-time LTI system with the property that if the input is

$$x(n) = \left(\frac{1}{2}\right)^n \mathrm{u}(n) - \frac{1}{4}\left(\frac{1}{2}\right)^n \mathrm{u}(n-1)$$

   then the output is

$$y(n) = \left(\frac{1}{3}\right)^n \mathrm{u}(n)$$

   (a) Determine the impulse response $h(n)$ and the system transfer function $H(z)$ of a system that satisfies these conditions.

   (b) Find the difference equation that characterizes this system.

   (c) Determine a realization of the system that requires the minimum possible amount of memory.

   (d) Determine if the system is stable.