ECE-486 Spring 2009, Feb 25, 2009
Test 1: 2 Hours; Closed book; No calculators

Name : ___________________________________

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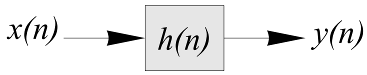
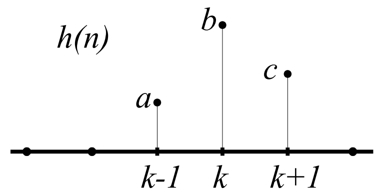
1. Find the time-domain signal $x(n)$ associated with the given transform.

$$X(z) = \frac{2z}{2z+1} + \frac{2z+2}{(z-2)(z-\frac{1}{2})} + \frac{3z^2 + (1/\sqrt{2})z}{z^2 - (2/\sqrt{2})z + 1} \qquad 1 < |z| < 2$$

2. A discrete-time time-invariant linear system has real impulse response as illustrated below, where k is an unknown integer, and a, b and c are unknown real numbers.





It is known that the system satisfies the following properties:

(a) The DTFT of h(n) has the form $H_R(f)e^{-j2\pi f}$, where $H_R(f)$ is a real function.

(b) If $x(n) = (-1)^n$ for all n, then $y(n) = 0$.

(c) If $x(n) = \left(\frac{1}{2}\right)^n \mathrm{u}(n)$, then $y(2) = \frac{9}{2}$.

Determine the values k, a, b, and c. Sketch the impulse response $h(n)$. Find and sketch the real-valued transfer function $H_R(f)$.

3. A causal discrete-time system is described by the difference equation

$$y(n) - \frac{2}{3}y(n-1) + \frac{1}{12}y(n-2) = 2x(n) - \frac{2}{3}x(n-1)$$

The system is known to have impulse response

$$h(n) = \begin{cases} \left(\frac{1}{2}\right)^n + \left(\frac{1}{6}\right)^n & n \geq 0 \\ 0 & \text{elsewhere} \end{cases}$$

(a) Assume that $y(-1) = -12$, $y(-2) = 24$, and $x(n) = 0$. Provide a z-transform which could be used to determine $y(n)$ for $n \geq 0$. (No need to actually evaluate the inverse transform!)

(b) For another set of initial conditions (different from those of part 3a), the system output for $x(n) = 0$ is projected to be

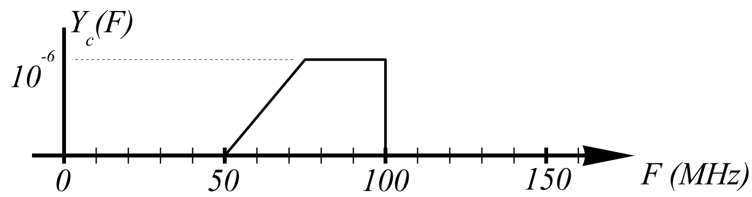
$$y(n) = 5\left(\frac{1}{2}\right)^n u(n) + 2\left(\frac{1}{6}\right)^n u(n)$$

For these initial conditions, we wish to drive the system input at $n = 0$ and $n = 1$ *only* with the goal of bringing the system to rest for $n > 1$. Find input values α and β so that the input

$$x(n) = \begin{cases} \alpha & n = 0 \\ \beta & n = 1 \\ 0 & \text{elsewhere} \end{cases}$$

will result in a system output that satisfies $y(n) = 0$ for $n \geq 2$.

4. A continuous-time real signal $y_c(t)$ has a real Fourier transform as indicated in the diagram below. The signal is sampled to form the discrete-time sequence $x(n) = y_c(nT_s)$ using the sampling frequency $F_s = 1/T_s$.



(a) Carefully draw the DTFT of $x(n)$ for $F_s = 50$ MHz.

(b) Carefully draw the DTFT of $x(n)$ for $F_s = 100$ MHz.

(c) Carefully draw the DTFT of $x(n)$ for $F_s = 300$ MHz.

(d) For the case $F_s = 300$ MHz, identify all frequencies below 1 GHz which would cause distortion of the above signal if not removed prior to sampling.

(e) For the case $F_s = 300$ MHz, let $s(n) = (j)^n x(n)$. Carefully sketch the DTFT of $s(n)$.

Extra Page For Problem 4
(In case your pictures are as large as mine)

5. A causal linear time-invariant system has poles at $z = \frac{3}{4}$ and $z = -\frac{1}{2}$, and zeros at $z = 1.5e^{j\pi/3}$ and $z = 1.5e^{-j\pi/3}$. The system has a DC gain of 10.

   (a) Determine the transfer function $H(z)$ of the system. Provide the ROC of you result. Write the transfer function $H(z)$ using only real polynomial coefficients.

   (b) Write down the functional form of the impulse response of the system. (No need to evaluate gain constants for each function... but your solution should include all functional forms that are present in the impulse response. e.g. "$Ag_1(n) + Bg_2(n) + \ldots$", where A, B, $\ldots$ are left unspecified.

   (c) Is this system stable? (Justify)

   (d) Describe the system using a constant-coefficient difference equation, and draw a block diagram of a system implementation which requires a minimum amount of memory.