ECE-486 Test 1, March 1, 2006
Two Hours, Closed Book, No Calculators

1. Evaluate the inverse z-transform of

$$X(z) = 5 + \frac{1}{z - 0.5} + \frac{2z}{z+1} + \left(\frac{z}{z - 0.25}\right)\left(\frac{5z+1}{z+2}\right) \qquad 0.5 < |z| < 1.0$$

2. Evaluate the z-transforms of the following signals. Give "closed-form" results (with a finite number of terms), and indicate the ROC for your expressions.

   (a) $\{\ldots, 0, 0, 0, 1, 2, \underset{\uparrow}{3}, 4, 0, 0, \ldots\}$

   (b) $5\cos(2\pi(0.3)n + \pi/6)u(n)$

   (c)

$$x(n) = \begin{cases} 2(0.5)^{|n-2|} & n \geq 0 \\ 0 & \text{elsewhere} \end{cases}$$

3. Each value of the "Fibonacci Sequence" $g(n)$ is found by adding the two previous values. By definition, $g(0) = g(1) = 1$, so the sequence is given by

$$\underset{\uparrow}{1}, 1, 2, 3, 5, 8, 13, 21, 34, \ldots$$

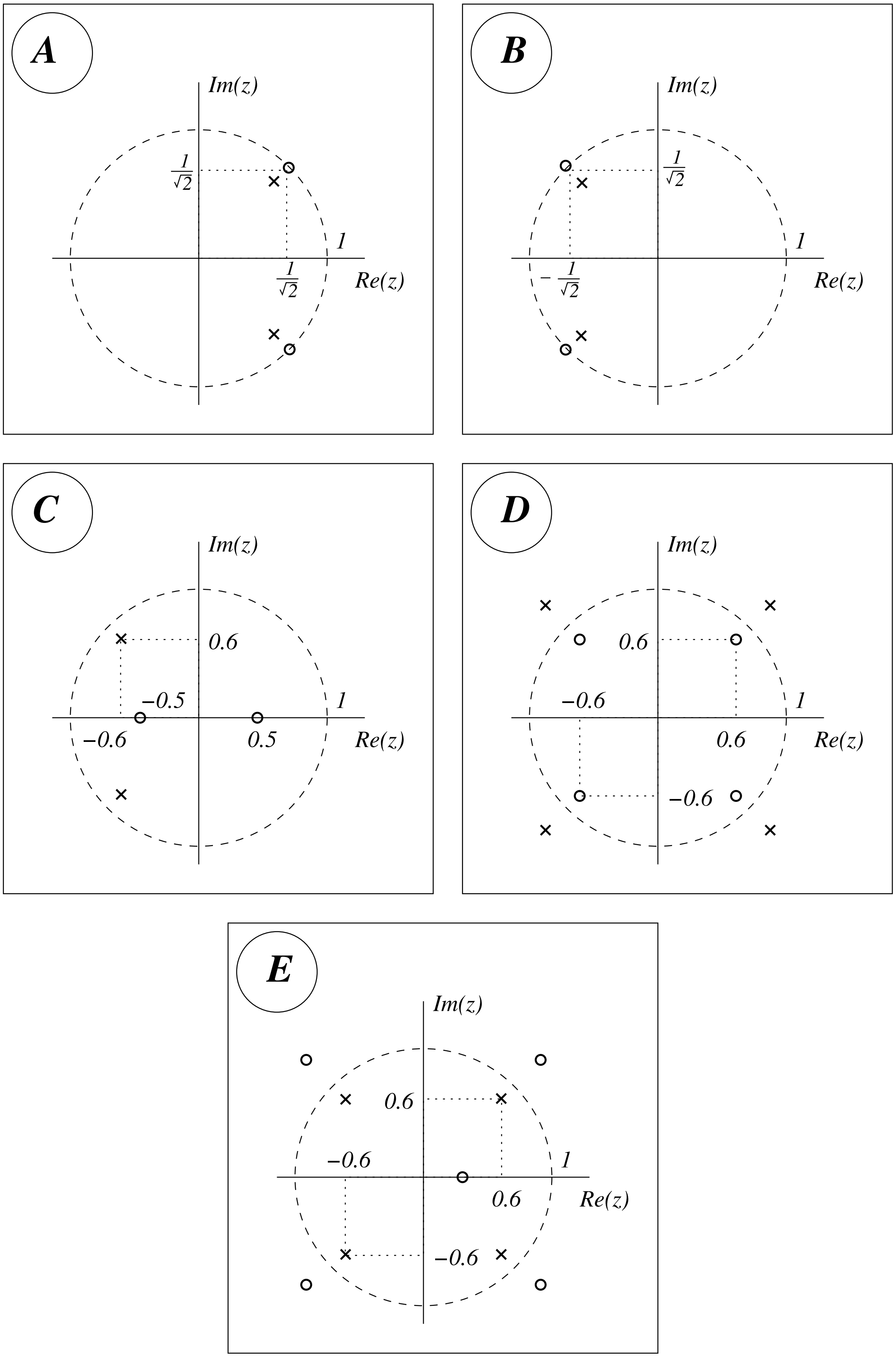
   (a) Give a difference equations which describes the generation of the sequence.

   (b) Provide initial conditions that result in the desired values of $g(0) = g(1) = 1$ (for no system input).

   (c) Find a "closed form" formula for calculating $g(n)$.

   (d) Is your system stable? (Justify)

4. Consider the five pole-zero plots below. Each plot corresponds to a discrete-time LTI system with real impulse response.

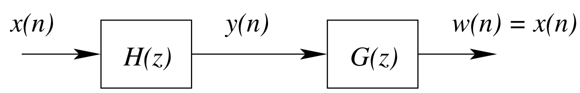


(a) (Problem 4 continued) Which plot(s) can have a ROC so that it corresponds to a causal and stable system?

Which Plot(s)?

Brief Explanation:

(b) Consider the following block diagram



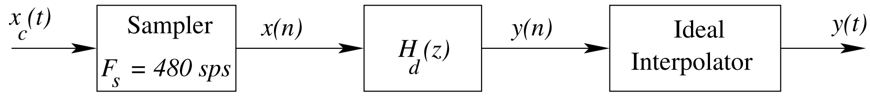
$H(z)$ is described by one or more of the pole-zero plots A-E. $G(z)$, which does not correspond to any of the pole-zero plots, is a system such that $w(n) = x(n)$. Which plot(s) correspond to $H(z)$ such that both $H(z)$ and $G(z)$ are causal and stable?

Which Plot(s)?

Brief Explanation:

(Problem 4 continued)

(c) Now consider the following system



Assume that $x_c(t)$ is sufficiently band-limited so that the Nyquist criterion is met. $x_c(t)$ includes a desired signal $s(t)$ and a 60 Hz sinusoidal interference.

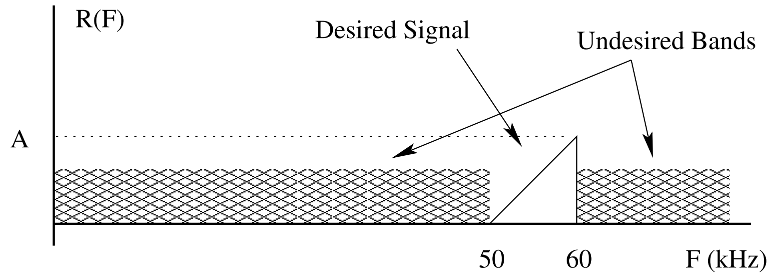
$$x_c(t) = s(t) + A\cos(2\pi \cdot 60t)$$

Which pole-zero plot corresponds to the best choice for $H_d(z)$ such that $|Y(j\Omega)|$, the magnitude of the Fourier transform of the overall output $y(t)$ is approximately equal to $|S(j\Omega)|$, the magnitude of the Fourier transform of $s(t)$?

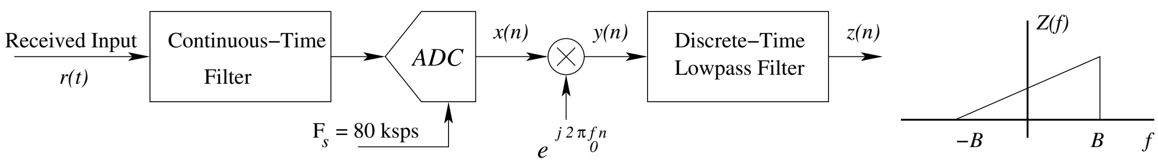
Which Plot?

Brief Explanation:

5. A continuous-time receiver input $r(t)$ has spectrum as illustrated below. A "software receiver" is to be designed which demodulates the signal in the 50-60 kHz band of frequencies.



A block diagram of the receiver is illustrated below, along with the desired output spectrum.



(a) Specify the required passband and stopband for the continuous-time filter. Make the transition bands (between the passband and stopband edges) as large as possible.

(b) Sketch and carefully label $X(f)$ for $-1 < f < 1$.

(c) Give the required value for f_0.

(d) Sketch and carefully label $Y(f)$.

(e) Give the value of B (the passband edge for the discrete-time lowpass filter).

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