ECE-486 Test 1, March 16, 2005
Two Hours, Closed Book, No Calculators

1. Find the z-transforms of the following:

   (a) $x(n) = \{\ldots, 0,\ 0,\ 1,\ \underset{\uparrow}{1},\ 1,\ 0,\ 0, \ldots\}$

   (b) $y(n) = \{\ldots, -1,\ 1,\ -1,\ \underset{\uparrow}{1},\ 0,\ 0,\ 0, \ldots\}$

   (c) u$(-n)$

   (d) $(1/2)^{|n|}$

2. Find the inverse z-transform of the following:

$$\frac{z}{z-1} + \frac{z(z+4)}{z-2} - \frac{5}{z-3} + \frac{z^4}{z^2-16} \qquad 2 < |z| < 3$$

3. A discrete-time causal system is described by

$$y(n) - 5y(n-1) = 2.4x(n).$$

(a) Find the transfer function of the system (specify the ROC of your result).

(b) Draw the pole-zero diagram for this system.

(c) Is this a BIBO stable system?

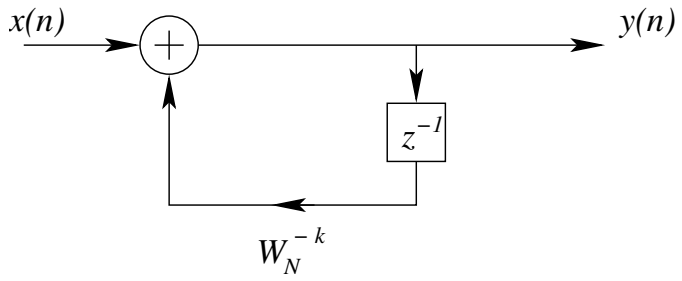
(d) Find the impulse response of the system.

(e) Find the system output for $x(n) = u(n)$ and $y(-1) = 3$.

4. A continuous-time signal $x_c(t)$ is sampled using a sample frequency of 10 ksps, giving a discrete-time signal $x(n)$. A 1000-point DFT of $x(n)$ is calculated, showing large values at DFT indexes $k = 95$ and $k = 905$.

   Identify all continuous-time frequencies below 50 kHz which may have been responsible for the large components in the DFT.

5. Let $W_N = e^{-j2\pi/N}$ be a complex constant, and consider the system illustrated below.



(a) Find the transfer function of the system (indicate the ROC).

(b) Find the system impulse response.

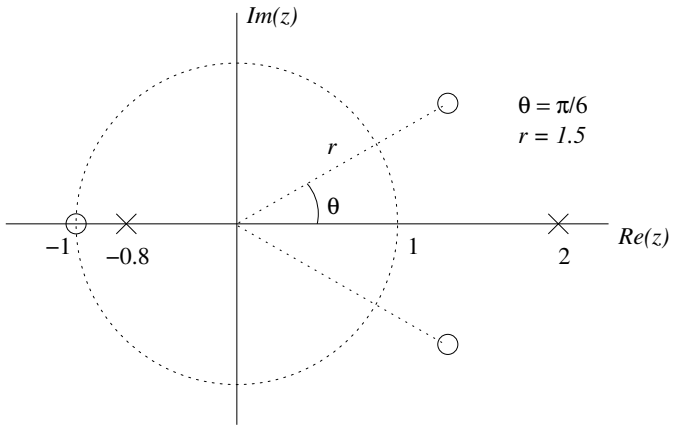
(c) Is this a B.I.B.O. stable system? (justify)

(d) Is this a causal system? (justify)

(e) Is this an FIR or IIR system? (justify)

(f) Now assume that $x(n) = 0$ for $n < 0$ and $n \geq N$. Show that at time $n = N$, the system output $y(N)$ provides the kth DFT sample $X(k)$.

6. The pole-zero diagram for a discrete-time system is illustrated below.



(a) Determine the ROC of the system function $H(z)$ if the system is known to be stable.

(b) Is it possible for the given pole-zero plot to correspond to a causal and stable system? (Justify)

(c) How many possible systems can be associated with this pole-zero pattern?

(d) Determine the system transfer function if it is given that the DC gain of the system is 1 (specify the corresponding ROC).

7. Let $x_c(t) = \cos(2\pi(9 \times 10^3)t)$ is sampled using a sample frequency of 48 ksps to obtain the discrete-time signal $x(n)$. Give equations for and sketch the following. Be careful to distinguish between Kronecker delta functions and Dirac delta-functions. Label your plots carefully.

   (a) The continuous-time spectrum $X_c(F)$.

   (b) The DTFT of x(n).

   (c) The 16-point DFT of x(n).