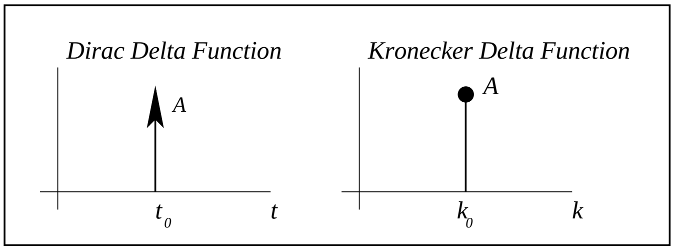
ECE486 Test 2, In Class Portion

April 22, 2004. One Hour
Closed Book, Closed Notes, No Calculators or Computers

In all of your plots, please be sure to distinguish between the Dirac delta function (shown on the left, with zero width, infinite height, and area labeled next to the arrow head), and the Kronecker delta function (shown on the right, with finite height labeled next to the dot).



All frequency-domain plots should clearly label the frequencies and amplitudes of major peaks.

The following transform pairs may (or may not) be helpful for the in-class portion of the test

$$r(n) = \begin{cases} 1 & n = 0, \ldots, M-1 \\ 0 & \text{elsewhere} \end{cases} \qquad R(f) = \frac{\sin(M\pi f)}{\pi f} \exp\left(-j\pi(M-1)\right)$$

1. Define $x(n) = A\cos(2\pi f_0 n)$
   (a) Write down and plot the DTFT of $x(n)$.
   (b) Write down and plot the N-point DFT of $x(n)$ (No credit for just copying the generic DFT formula... Give the transorm result for this $x(n)$). Give your equations in terms of the parameters f_0, A, and N.
   (c) Explain how your results would change if the sequence $x(n)$ were multiplied by an N-point window function $w(n)$ prior to calculation of the DFT. How would the height of the major peaks in the DFT change?

2. The signal $x(n)$ has five non-zero terms as given by

$$x(n) = \{\ldots, 0, 0, x(-3), x(-2), x(-1), \underset{\uparrow}{x(0)}, x(1), 0, \ldots\}.$$

Let $h(n)$ be the (non-causal) impulse response with seven non-zero terms as given by

$$h(n) = \{\ldots, 0, 0, h(-3), h(-2), h(-1), \underset{\uparrow}{h(0)}, h(1), h(2), h(3), 0, 0, \ldots\}.$$

For this problem, we wish to calculate the convolution $y(n) = x(n) * h(n)$ using N-point FFTs. That is, we'll define N-point arrays $x_1(n)$ and $h_1(n)$ initialized with the samples $x(n)$ and $h(n)$ respectively. An N-sample array $y_1(n)$ is then calculated using the following procedure:

$$\begin{aligned} X_1(k) &= N\text{-point DFT of } x_1(n) \\ H_1(k) &= N\text{-point DFT of } h_1(n) \\ Y_1(k) &= X_1(k)H_1(k) \\ y_1(n) &= N\text{-point inverse DFT of } Y_1(k) \end{aligned}$$

Values of the linear convolution $y(n) = x(n) * h(n)$ are to be obtained from the array $y_1(n)$.

(a) Find the minimum value of N for which the above procedure will work.

(b) Now assume the 16-point FFTs are to be used. Fill in the tables below, showing how to initialize the arrays $x_1(n)$ and $h_1(n)$, and show how to obtain the values of $y(n)$ from the array $y_1(n)$

n	$x_1(n)$
0	
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	
13	
14	
15	

n	$h_1(n)$
0	
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	
13	
14	
15	

n	$y_1(n)$
0	
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	
13	
14	
15	

3. Let $h(n)$ a linear phase FIR filter with an odd number of non-zero coefficients. Show that if $H(z)$ has a zero at $z = z_0$, then it must also have a zero at $z = 1/z_0$.