ECE-486 Test 2
May 1, 2003 — In Class Portion
Closed Book, No Calculators

When you are finished with this portion of the exam, turn it in and go to work on the computer portion. After you turn this exam in, you may not return to this part of the test.

1. Let $x(n)$ and $h(n)$ be finite length sequences defined by

$$\begin{aligned} x(n) &= \{\ldots, 0, 3, 4, 5, \underset{\uparrow}{6}, 7, 0, 0, \ldots\} \\ h(n) &= \{\ldots, 0, \underset{\uparrow}{1}, 2, 3, 0, 0, \ldots\} \end{aligned}$$

This problem concerns the calculation of the linear convolution of $x(n)$ and $h(n)$ using the DFT. Assume that values of $x(n)$ and $h(n)$ are stored in the N sample array $x_1(n)$ and $h_1(n)$ respectively. An N-sample array $y_1(n)$ is then calculated using the following procedure:

$$\begin{aligned} X_1(k) &= N\text{-point DFT of } x_1(n) \\ H_1(k) &= N\text{-point DFT of } h_1(n) \\ Y_1(k) &= X_1(k)H_1(k) \\ y_1(n) &= N\text{-point inverse DFT of } Y_1(k) \end{aligned}$$

Values of the linear convolution $y(n) = x(n) * h(n)$ are to be obtained from the array $y_1(n)$.

(a) Find the minimum value of N for which the above procedure will work.

(b) Using a valid value of N which is a power of two, show how the values of $x_1(n)$ and $h_1(n)$ should be initialized for $n = 0, 1, 2, \ldots, N-1$.

(c) Show how to obtain the linear convolution results $y(n)$ from the array $y_1(n)$.

2. A communications system demodulator involves multiplying a signal $x(t)$ by a 15 kHz cosine

$$y_c(t) = x_c(t)\cos(2\pi(15 \times 10^3)t)$$

The system is to be implemented using a signal processor which uses a sampling rate of $F_s = 48$ ksps. Let $x(n)$ and $y(n)$ be samples of the continuous-time signals $x_c(t)$ and $y_c(t)$ respectively.

$$y(n) = x(n)\cos(2\pi f_o n)$$

(a) Find the appropriate value of f_0.

(b) In lab, you implemented a similar mixer using a lookup table to store values of the trigonometric function cos(). For this case, what is the minimum length of the required lookup table? Show how values to be stored in the lookup table should be calculated.

3. A linear phase FIR filter has impulse response

$$h(n) = \{\ldots, 0, \underset{\uparrow}{1}, -1, 2, -1, 1, 0, 0, \ldots\}$$

This FIR filter has zeros located at $j, e^{j\pi/3}, -j$, and $e^{-j\pi/3}$.

(a) What is the delay associated with this linear phase filter?

(b) Write down the real-valued transfer function and the corresponding impulse response for this filter.

(c) Show how this filter could be implemented using a cascade of second-order systems. Give the transfer functions and impulse responses of each of the second order systems. Are your second order systems linear phase? (Justify)

4. How is the magnitude response of an elliptical lowpass filter different from that of a Butterworth lowpass filter?

5. Why would a designer prefer to implement a 10th order IIR filter as a cascade of five second-order filters?