

ECE-486 Test 1
March 27, 2003 Closed Book, No Calculators

1. Evaluate the inverse z-transform of

$$X(z) = \frac{3z^4 + 3z^3}{(z + \frac{1}{2})(z + 2)} + z^{-3} + \frac{1}{z - \frac{1}{2}} - \frac{z}{z - 3} \quad \frac{1}{2} < |z| < 2$$

2. True or false? ... *Every FIR filter is stable...*
(Justify your response.)

3. The trapezoidal integration formula is described by

$$y(n) = y(n-1) + \frac{1}{2}(x(n) + x(n-1))$$

- (a) Find the transfer function $H(\omega)$ for this system.
- (b) Show that the system approximates an integrator for small ω .
- (c) Sketch the pole-zero diagram for this system.
- (d) Is this a stable system? (justify).
- (e) Find and sketch the impulse response of the system.
- (f) Is this a FIR or an IIR filter? (justify)

4. An audio continuous time signal $x_c(t)$ with bandwidth 500 Hz is sampled using a sample rate of 1000 samples/sec, The output sample sequence $x(n)$ is to be processed using the filter

$$y(n) = b_0x(n) + b_1x(n - 1) + b_2x(n - 2).$$

The filter is to be designed such that it has DC gain of one, and completely rejects signal components corresponding to the (continuous-time) frequency 100 Hz.

Find (but don't solve!) a set of three independent linear equations which could be solved to determine the required filter coefficients b_0 , b_1 , and b_2 .

5. A discrete-time system has impulse response given by

$$h(n) = \{\dots, 0, \underset{\uparrow}{1}, 0, 1, 0, 0, \dots\}$$

- (a) Find the transfer function $H(\omega)$ for this system, and sketch the magnitude and phase response of the system.
- (b) Find the system output (closed form) when the system input is given by $x(n) = 5 \cos(\frac{\pi}{3}n)$
- (c) Let $H(k)$ be the 8-point DFT of $h(n)$. Sketch $|H(k)|$.

6. A continuous-time signal $x_c(t)$ has Fourier transform as illustrated below. The signal is to be sampled and processed by the system shown.

- Sketch the discrete-time Fourier transform of the sampled sequence $x(n)$.
- Find the relationship between $Y(f)$ and $X(f)$. Sketch $Y(f)$.

