

**ECE-486 Test 1**  
**March 28, 2002 Closed Book, No Calculators**

1. Evaluate the following. Give your answers as a single real number, or (if complex) in the form  $x + jy$ , where  $x$  and  $y$  are real.

(a)  $\int_0^T e^{j2\pi nt/T} dt$  ( $n$  is an integer)

(b)  $\text{Re}\{5e^{j\pi/7}e^{j\omega_0 t}\}$

(c)  $\frac{2e^{j\pi/3}}{3e^{-j\pi/6}}$

2. Find the z-transforms of the following signals. Be sure to specify the region of convergence of your answer.

(a)  $0.3 \cos((0.1)n + \pi/3)u(n)$

(b)  $\{\dots, 0, 0, 1, \underset{\uparrow}{2}, 3, 4, 5, 0, 0, 0, \dots\}$

3. Find the inverse z-transforms of the following signals.

(a)

$$X(z) = \frac{2}{z(z - 1/4)} + \frac{3z}{z + 1/2} + \frac{5}{z - 2} + 5z \quad 1/2 < |z| < 2.$$

(b)

$$X(z) = \frac{5z^2 + 2z}{(z + 1/2)(z + 1/3)} \quad |z| > 1/2.$$

4. A time-invariant linear system has transfer function

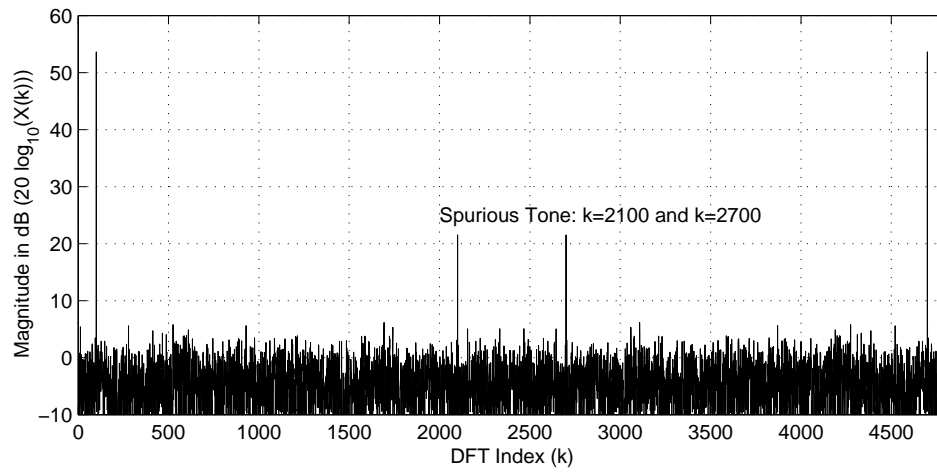
$$H(z) = \frac{z^2 - 0.6619z}{z^2 - 1.3237z + 0.9025} = \frac{z(z - 0.6619)}{(z - 0.95e^{j0.8})(z - 0.95e^{-j0.8})}$$

The impulse response of the system is

$$h(n) = (0.95)^n \cos(0.8n)u(n).$$

- (a) Find the response of the system to input  $x(n) = e^{j(0.3)n}$ .
- (b) Find the response of the system to input  $\{\dots, 0, 0, 1, 2, 3, 0, 0, 0, \dots\}$ .
- (c) Sketch the pole-zero diagram for this system. Include the unit circle on your sketch.
- (d) Is this system causal? (justify)
- (e) Is this a stable system? (justify)
- (f) Give a difference equation which could be used to implement this system.
- (g) Draw a block diagram showing a direct-form II implementation of the system.

5. An audio amplifier is being tested by driving the amplifier with a 1 kHz test signal and sampling the output using a sampling frequency of 48 ksp/s. (The signal is *not* filtered prior to sampling.) A buffer of 4800 samples is collected, and used to calculate a DFT, which is plotted below. The plot shows the presence of undesired terms at DFT indices 2100 and 2700. Identify all possible continuous time frequencies below 100 kHz which could have produced these terms.



6. Consider a lowpass filter

$$y(n) = ay(n-1) + bx(n) \quad 0 < a < 1$$

- (a) Determine  $b$  (in terms of  $a$ ) so that the DC gain of the filter is one.
- (b) Find an expression (in terms of  $a$ ) for the 3-dB bandwidth  $\omega_3$  of the normalized filter of part 6a