ECE-486 Test 1
March 6, 2001

1. A causal IIR filter is described by the difference equation

$$y(n) - 0.9y(n-1) + 0.91y(n-2) = 2x(n) - 0.9x(n-1)$$

   (a) Is this system stable? (Justify)

   (b) Find $Y^+(z)$ for $x(n) = (0.5)^n u(n)$ and $y(-1) = 1$, $y(-2) = 2$.

   (c) Find the transfer function $H(z)$ for the system.

2. A designer wishes to approximate the derivative of a signal $x_a(t)$ by sampling the signal using a sample rate of $F_s = 48$ ksps, and calculating

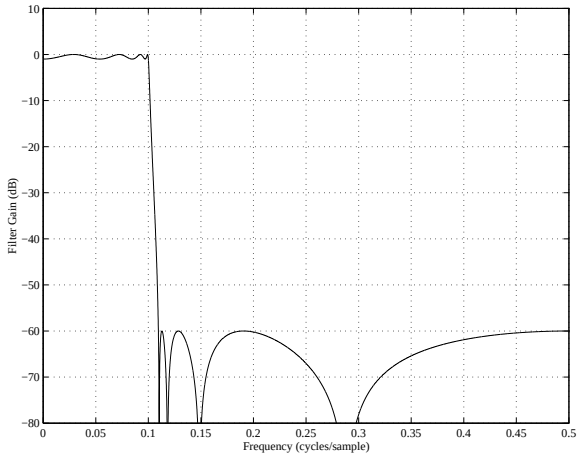
$$y(n) = \frac{1}{2T_s}(x(n+1) - x(n-1))$$

Write down the impulse response $h(n)$, the transfer function $H(z)$. and find and sketch the magnitude of the frequency response $|H(f)|$.

3. The plot below shows the magnitude response of a real FIR filter with impulse response $h_1(n)$. A second filter is designed by setting

$$\begin{aligned} h_2(n) &= (-1)^n h_1(n) \\ &= \begin{cases} h_1(n) & n = 0, 2, 4, 6, \ldots \\ -h_1(n) & n = 1, 3, 5, \ldots \end{cases} \end{aligned}$$

Sketch the magnitude response of the second filter ($|H_2(f)|$) for $-0.5 < f < 0.5$.



4. Find a difference equation which describes a system with impulse response $h(n) = \cos(0.72n)u(n)$. Make your system have the lowest order possible. Draw a block diagram showing the direct-form II implementation of your system.

5. Is the system of Problem 4 stable? Justify your answer.

6. Write down the inverse z-transforms of the following:

(a)
$$\frac{1}{z-1} \qquad |z| > 1$$

(b)
$$5z^{-2} + 4z^{-1} + 3 + 2z + z^2 \qquad 0 < |z| < \infty$$

(c)
$$\frac{z^2}{(z-0.5)(z-0.75)} \qquad 0.5 < |z| < 0.75$$

(d)
$$\frac{2z^2 - 0.8z}{z^2 - 0.9z + 0.81} \qquad |z| < 0.9$$

7. A distorted (and noisy) sinusoidal signal with frequency 1.9 MHz is sampled using a sampling frequency of 4.096 Msps to obtain a buffer of samples $x(n)$ for $n = 0, 1, \ldots, 1023$. A 1024-point DFT has been calculated to obtain the plot shown below.

   (a) Circle and label the component of the output spectrum which is associated with the sinusoidal test signal. Give the exact DFT/FFT index associated with this term.

   (b) Repeat the above for the 9th harmonic of the test signal.