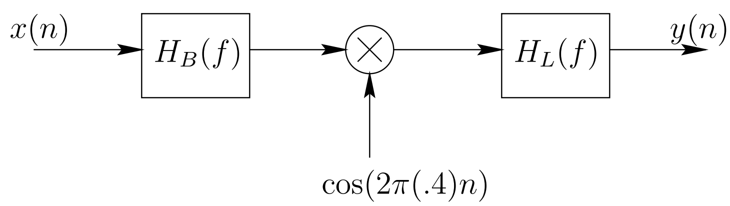
ECE-486 Test 2
Spring 2000
Due 11:00 AM, May 5

- Do your own work. Do not discuss the problems with any other person. Do not talk about the test with any other student until after the due-time. Don't even say things like "Number 2 seems hard", or "Did number 3 seem trivial to you?". Do not incur the wrath of the instructor! Be an honest person.
- Unless otherwise stated, you may use MATLAB or MATHCAD for any of the problems. *Please do so*. Numerical errors will be harshly graded. Check your answers!
- For each problem, make (labeled) plots illustrating that your filter designs have met the stated specifications.

1. Design a 6th order bandpass filter with passband $0.15 \leq f \leq 0.20$ and passband gain between -0.5 dB and 0 dB. Set the minimum stopband attenuation to 50 dB, and make the transition between passband and stopband as narrow as possible.

   Specify your solution as a cascade of three second-order sections, all with real filter coefficients.

2. Consider the design of a single sideband (SSB) demodulator which is implemented using a sampling frequency of 100 ksps. The signal of interest is the upper sideband of a 3 kHz bandwidth audio signal which has been modulated using a carrier frequency of 40 kHz ($f_c = 40/100 = 0.4$). A block diagram of the SSB demodulator is given below, where $H_B(f)$ denotes a bandpass filter response, and $H_L(f)$ denotes a lowpass filter response.



   Assume that audio frequencies in the range $300\text{ Hz} < F < 3$ kHz for the modulated message are to be preserved by the SSB demodulator, with no more than ± 0.2 dB variation across the band. Also assume that other (undesired) signals may be present below 39.7 kHz or above 44 kHz. These signals should be attenuated by at least 60 dB. Find reasonable design specifications for the bandpass and lowpass filters. Do not over-specify the filters! Specify the (discrete-time) frequency passband and stopband for each filter, and include the desired gain in each of the bands.

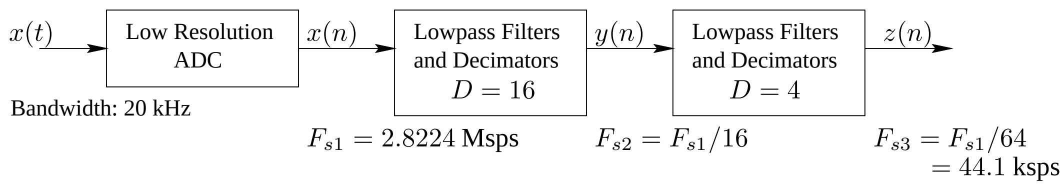
3. Design minimum order FIR and IIR lowpass filters which meet the following specifications.

Passband:	$0 < \lvert f \rvert < 0.1$	Gain $= 20$ dB ± 0.2 dB
Stopband:	$\lvert f \rvert > 0.15$	Gain ≤ -60 dB

   Indicate the order of your filter, the design method used (including how you selected filter design parameters) and give plots showing that you've met the desired specs. Evaluate the number of multiplications required to calculate each filter output.

4. A *sigma-delta analog-to-digital converter* ($\Sigma\Delta$ ADC) operates by collecting low-resolution samples of an analog signal at a sampling frequency which is much higher than the Nyquist sampling frequency. The samples are then digitally filtered to remove most of the noise which was introduced by the sampler. The sampling frequency is reduced as the signal is filtered to create high-resolution samples at the reduced rate. Many modern audio-frequency CMOS ADCs are $\Sigma\Delta$ converters.

   For this problem, consider a $\Sigma\Delta$ ADC which is to be designed to sample input signals $x(t)$ with 20 kHz bandwidth. A low-resolution (about 3-bits) CMOS sampler is constructed which operates at a sample frequency of $F_{s1} = 2.8224$ MHz. The output sequence $x(n)$ from this sampler must be digitally filtered and decimated, forming a high-resolution (16-bit) sequence $z(n)$ at reduced sample rate $F_{s3} = F_{s1}/64 = 44.1$ ksps. The process is accomplished in two filter/decimation stages and is illustrated below. This problem involves the design of the filters needed to accomplish the decimation.

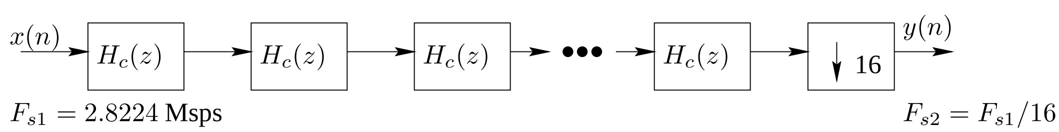


   (a) *First Decimation Stage: Comb filters*

   The first decimation stage must reduce the sampling frequency by a factor of 16. To simplify the required hardware at the high sample rate, multiple "comb" filters are to be used to accomplish the required filter operations. Each comb filter has transfer function

$$H_c(z) = \frac{1}{16}\frac{1 - z^{-16}}{1 - z^{-1}}$$

   The number of comb filters used depends upon the desired level of distortion which will be allowed to alias into the signal band ($|F| < 20$ kHz). The structure of the first decimation stage is illustrated below.



   Identify all discrete-time frequencies for the signal $x(n)$ which will alias into the desired signal band when the sample rate is reduced by 16.

   Find the number of comb filter stages required so that each of the above aliased bands is attenuated by at least 90 dB prior to the decimation.

   (b) *Second Decimation Stage: FIR Filter*

   The second decimation stage must reduce the sample rate by a factor of four, from 176.4 ksps to 44.1 ksps. An FIR filter using sample rate $F_{s2} = 176.4$ ksps is to be used to prevent aliasing in the second stage, and "equalize" the (non-flat) passband response of the first decimation stage. The filter is to be designed to equalize the $|F| < 20$ kHz band, while rejecting (by at least 90 dB) all frequencies above 24 kHz.

   The desired frequency frequency response for the FIR filter is given by

$$|H_2(f)| = \begin{cases} \left(\frac{16\sin(\pi f/16)}{\sin(\pi f)}\right)^8 & |f| < 0.11338 \\ 0 & |f| > 0.13605 \end{cases}$$

   Design a minimum-phase FIR filter which has gain within 0.1 dB of the desired response within the filter passband, and provides at least 90 dB stopband attenuation. Indicate the order of your filter, the design method used (including how you selected filter design parameters) and give plots showing that you've met the desired specs.