ECE-486 Test 1
October 20, 1999

1. (15 pts) A discrete-time system is described by the difference equation given below. Find $Y(z)$ for input $x(n) = 2\cos(0.1n)\mathrm{u}(n)$, and initial conditions $y(-1) = 4$, $y(-2) = 2$. (Don't solve for $y(n)$.)

$$y(n) + 3y(n-1) + 10y(n-2) = 0.5x(n-1) - x(n-2)$$

2. (10 pts) Let $h(n)$ denote the impulse response for the above system. Find $h(0)$, $h(1)$, and $h(2)$.

3. (15 pts) The transfer function of a Discrete-time system is given by

$$H(z) = \frac{-z(3z+5)}{(z-0.2)(z-3)} \quad 0.2 < |z| < 3$$

   (a) (8 pts) Find the impulse response for this system.

   (b) (7 pts) Is this system stable? (Defend your answer.)

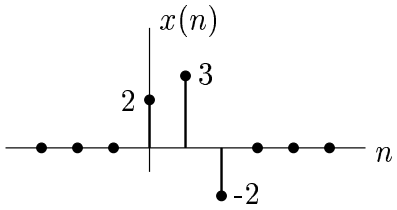
4. (10 pts) An analog waveform $x(t)$ is sampled at $f_s = 300$ kHz producing a sequence of samples. This sequence is filtered using a discrete-time filter with transfer function $H(z)$. The filter is designed such that $H(\omega)$ approximates the ideal bandpass characteristic

$$H(\omega) \approx \begin{cases} 1 & 0.1\pi < |\omega| < 0.4\pi \\ 0 & \text{elsewhere} \end{cases}$$

   The filter output samples are then used to drive a digital-to-analog converter, followed by an ideal low-pass filter with cutoff frequency 150 kHz. Find the band of frequencies which will be passed from $x(t)$ to the system output $y(t)$.

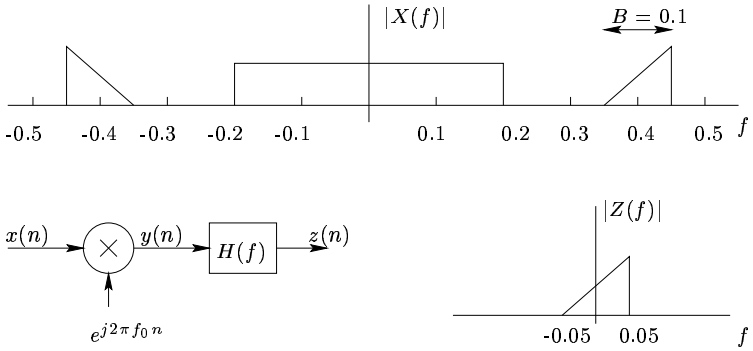
5. (20 pts) A discrete-time system has transfer function $H(z) = \dfrac{z^2 - 0.5z}{z^2 - z + 1}$ and impulse response $h(n) = \cos(n\pi/3)\mathrm{u}(n)$.

   (a) (10 pts) Find the system output (closed form) for zero initial conditions and input $x(n)$ illustrated below.



   (b) (10 pts) Find the system output (closed form) for zero initial conditions and input $x(n) = e^{j0.5n}$ (note that $x(n) \neq e^{j0.5n}\mathrm{u}(n)$) .

6. (18 pts) A block diagram for the front end of a digital receiver implementation is illustrated below. The input signal $x(n)$ contains a desired signal with bandwidth B centered at 0.4 cycles/sample. It also contains undesired signals for $|f| < 0.2$. The system is to be designed such that the output spectrum $|Z(f)|$ has the shape shown.



(a) (6 pts) Find the required value of f_0.

(b) (6 pts) Sketch the spectrum of the intermediate signal, $|Y(f)|$.

(c) (6 pts) Specify the required passband and stopband for the filter $H(f)$.

7. (12 pts) The pole-zero diagrams for two discrete-time filters are illustrated below. The plots also show the unit-circle to provide a scale reference. Make a rough plot of the filter gain (in dB) versus the input signal frequency for each of the filters. Show your filter gain for $0 < f < 0.5$.